Equilibration in parton transport theory?

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Outline

Introduction, some initial thoughts

Boltzmann transport theory

Equilibration studies, E_T evolution and elliptic flow

Conclusion

Theoretical description of heavy-ion collisions

Challenging situation:

- initial condition known fairly well
  except impact parameter and polarization (known statistically only)
- final state measured almost completely
- "canonical" theory QCD known for over three decades
  BUT at present uncomputable for heavy-ion collisions

Consequences:

- approximate, phenomenological models
- no single model can describe complete evolution
  instead, several models stapled together

Importance of equilibration

If equilibrium does get established:

- theory becomes simpler

  $\rightarrow$ fewer parameters needed to describe state
  $\rightarrow$ easier to compute evolution

- horizon effect

  $\rightarrow$ cannot learn details of initial nonequil. evolution from final state
  $\rightarrow$ initial condition becomes a model parameter unless initial nonequilibrium evolution can be computed

Is equilibrium established?

- Low-$p_\perp$ particle spectra seem to be consistent with equilibrium
  (Heinz et al, Xu et al)
- BUT real proof: show equilibrium is achieved and maintained

  $\Rightarrow$ requires nonequilibrium framework

Boltzmann transport theory

- simplest, covariant nonequilibrium theory
  - describes evolution of single particle phasespace distr. $f(x,p)$
  - can also be obtained from QCD under certain conditions
- dynamics governed by the mean free path: $\lambda(s,x) = 1/\sigma(s)n(x)$
- ideal for equilibration studies:
  $\lambda \to 0$ limit leads to equilibrium dynamics (ideal hydro)

Nonlinear parton transport equation:

$$p^\mu \partial_\mu f_i(x,\mathbf{p}) \;=\; \overbrace{S_i(x,\mathbf{p})}^{\text{source}} \;+\; \overbrace{C_i^{el.}[f](x,\mathbf{p})}^{2\to 2\ (\mathrm{ZPC,GCP,PSYCHE})} \;+\; \overbrace{C_i^{inel.}[f](x,\mathbf{p})}^{2\leftrightarrow 3\ (\mathrm{MPC})} + \ldots$$

$\Rightarrow$ **covariant numerical solutions only recently available**

(Pang, Zhang et al, D.M., Gyulassy, Vance et al, Cheng, Pratt)

Ideal hydrodynamics vs. Boltzmann transport

(Csernai, Stöcker, Rischke et al vs. Gyulassy, Zhang, M., Vance, Pratt et al)

Common features:

- **Lorentz covariance and conservation laws incorporated**
- **no wave phenomena, no particle correlations**
- **need to start from an intermediate stage**

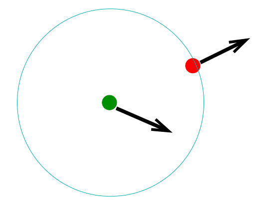
Differences:

- **hydro: limited to $\lambda = 0$, Boltzmann: can treat $\lambda \neq 0$**
- **hydro: ad-hoc freezeout, Boltzmann: natural freezeout**
- **hydro: can treat phase transitions, Boltzmann: cannot**
- **hydro: needs EOS, Boltzmann: needs transition probabilities**

Covariant solutions of Boltzmann transport

(nucl-th/0005051)

Usually: via the cascade technique $\approx$ billiard ball scattering
GCP, ZPC, MPC, PSYCHE



Problem: algorithm nonlocal $\Rightarrow$ action at a distance

leads to acausality (superluminal propagation) $\Delta v_{sig} \sim \frac{\sqrt{\sigma/\pi}}{\lambda}$

Solution: Pang's particle subdivision technique $(f \to lf,\ \sigma \to \sigma/l)$

- increases number of test particles by factor l
- reduces interaction range by factor $1/\sqrt{l} \Rightarrow \lambda$ stays same
- Lorentz covariance restored in the $l \to \infty$ limit

Note: nonlocal effects reduce elliptic flow and reheat the p_t spectrum (nucl-th/0107001)

For RHIC initial conditions, these are eliminated only if $l >\sim 200 - 1000$
(nucl-th/0005051, nucl-th/0104073)

$\Rightarrow$ real computational challenge

Equilibration studies via MPC

Idea:

- study whether equilibrium can be **maintained**
  i.e., start evolution from equilibrium

- focus on **quantities** that are driven by the **pressure**
  e.g., E_T and v_2

- **dissipative effects** modify pressure $p \neq p_{equil.}$
  $\Rightarrow$ deviations from equilibrium **reflected** in E_T work and v_2

General $\lambda \neq 0$ case falls **between** free streaming (nothing happens) and ideal case (maximum effect).

Question: how large are dissipative effects at RHIC?

Gluon E_T evolution at RHIC

Take ultrarelativistic gluon gas, $e = 3p$, initially in equilibrium

Initial conditions at RHIC: largely unknown

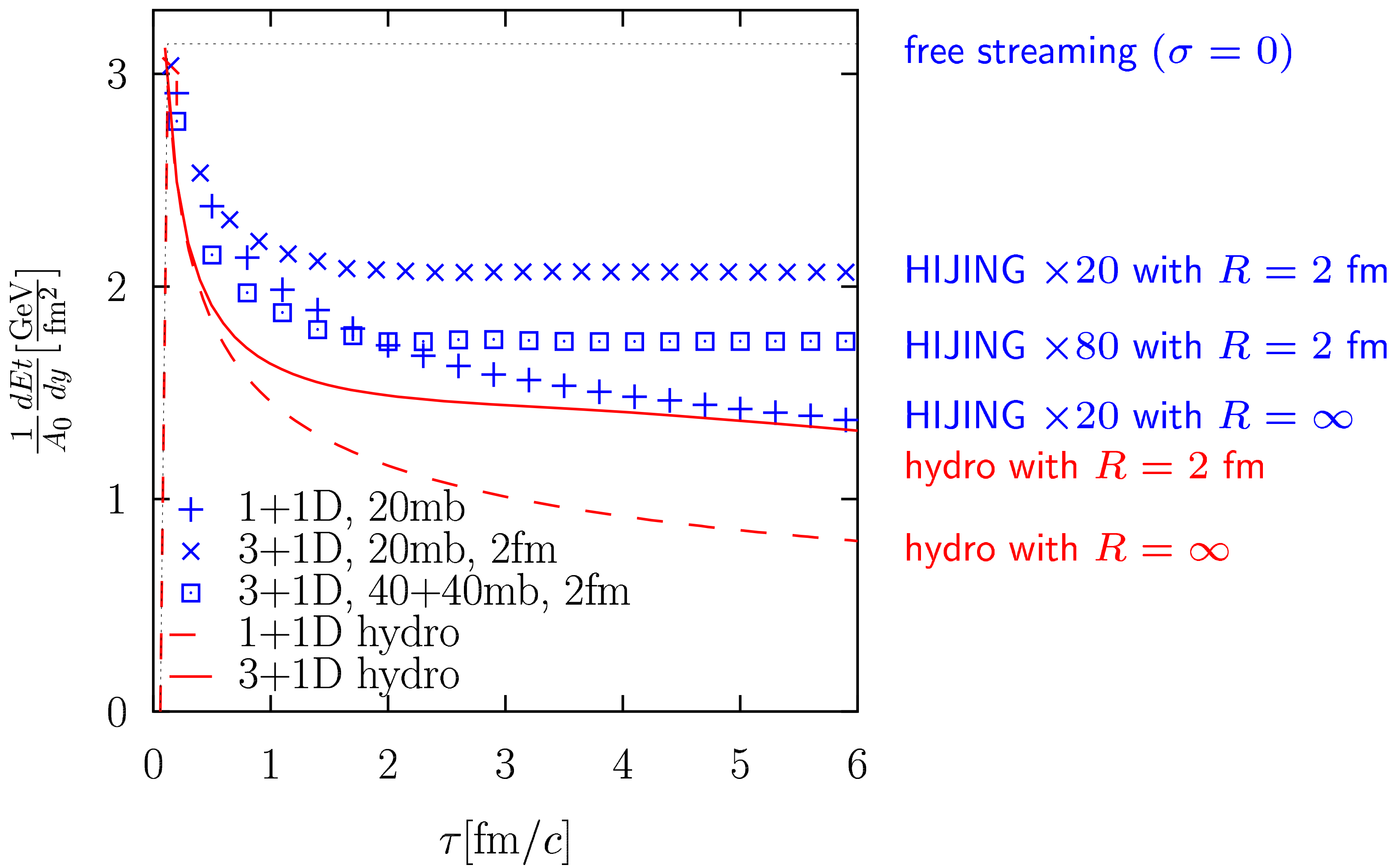
- **HIJING: $dN/dy_{glue} \sim 200$ for central 130 GeV/nucleon**
- **EKRT gluon saturation models: $dN/dy_{glue} \sim 1000$ for same energy**
- **take Bjorken cylinders with $R = 2$ fm and $R = \infty$, $\tau_0 = 0.1$fm$/c$**

Interactions:

- **take $2 \to 2$ interactions only**
- **due to scaling, only the product $\sigma dN/dy$ matters** (nucl-th/0005051)
  $\Rightarrow$ fix $dN/dy = 200$, vary $\sigma_{gg\to gg}$ (from pQCD $\sigma \sim 3$mb)
- **take isotropic differential cross section for maximum effect**

Important: no hydro freezeout assumptions needed for study

MPC vs hydro (1+1D and 3+1D)

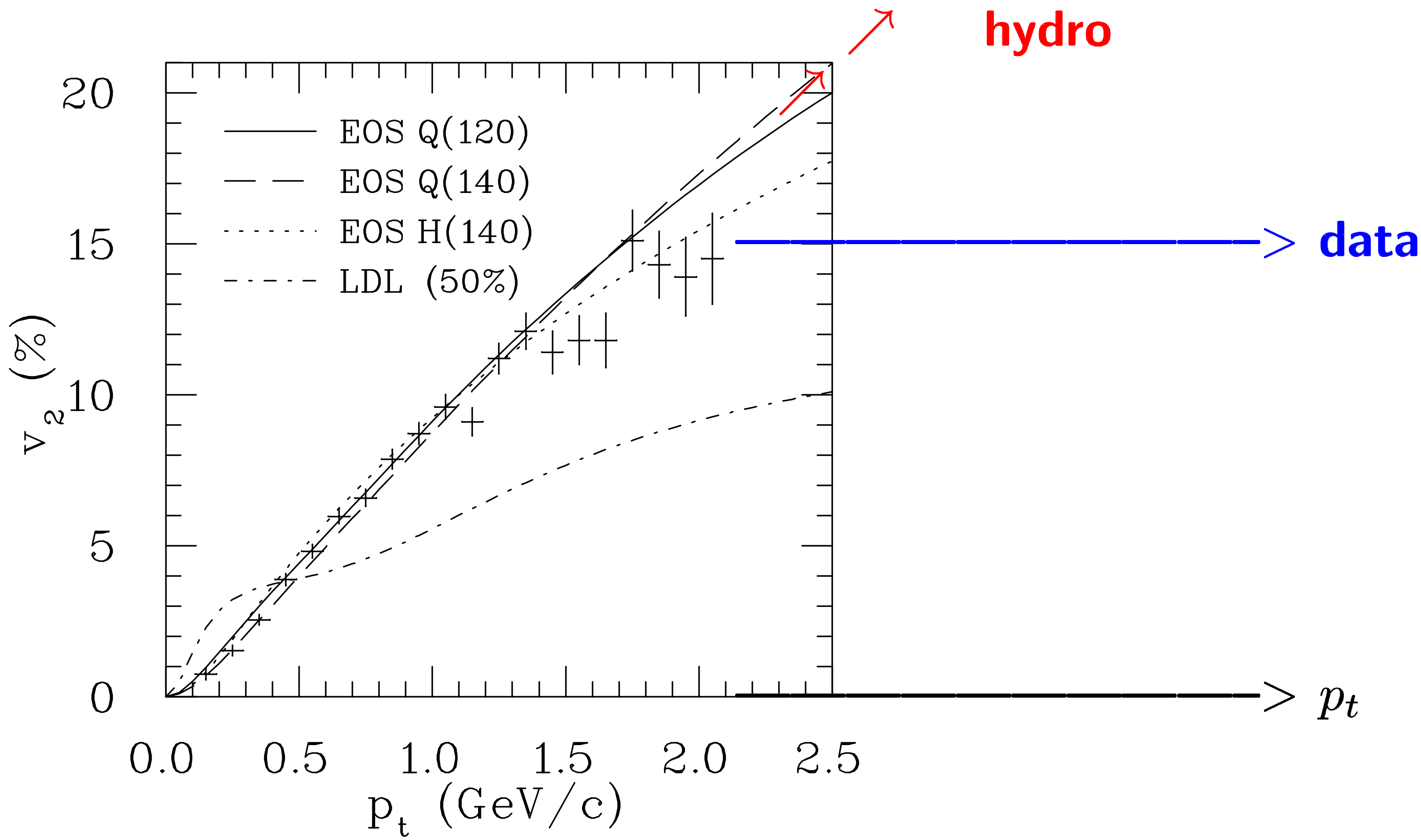


ideal hydro does more work even for $\sigma = 20$ mb ($\sigma_{pQCD} \approx 3$ mb)

$\Rightarrow$ **rates at RHIC cannot maintain equilibrium**
not even for extreme cross sections/densities

Saturation of elliptic flow at RHIC

- **Indicates nonequilibrium dynamics at RHIC**



- **Ideal hydrodynamics disagrees with data above $p_T \sim 1.5 - 2$ GeV independently of initial conditions and freezeout criteria**

  (Kolb et al, hep-ph/0012137)

Elliptic flow at RHIC from MPC

Initial condition at RHIC:

- $dN/dy_{glue} \sim 200 - 1000$ **for** $\sqrt{s} = 130$ **GeV (HIJING vs EKRT)**
- **take Bjorken tube with** T_{AB} **density profile,** $T = 0.7$**GeV,** $\tau_0 = 0.1$**fm**$/c$

Interactions:

- **screened** $gg \to gg$ **interactions only**
- **relevant parameter: transport opacity** $\chi \equiv N_{coll} \langle \sin^2 \theta_{cm} \rangle$

  $\chi \propto \langle \sin^2 \theta_{cm} \rangle \sigma dN/dy$ (nucl-th/0104073)

  $\to$ **think of** $\sigma = 3$ **mb FIXED,** dN/dy **VARIABLE**

Must model hadronization:

a) local parton-hadron duality (EKRT): $1g \to\approx 1\pi$

b) independent fragmentation: $g \to \pi$ **fragmentation function**
(hep-ph/9407347, Binnewies, Kniehl, Kramer)

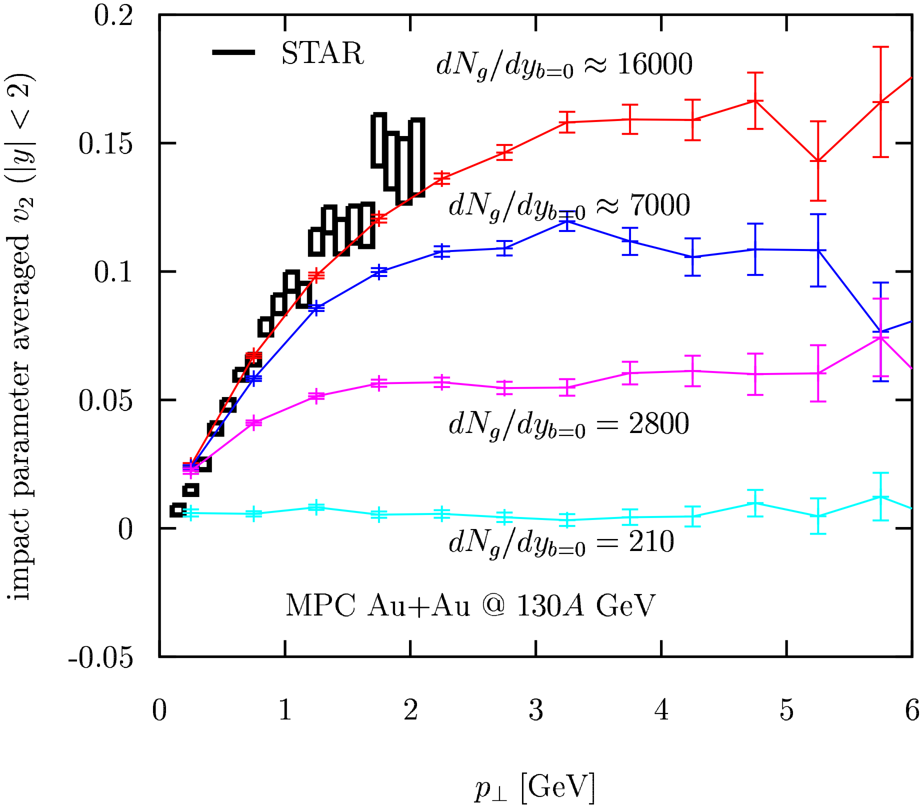
A) $\sigma_0 = 100$ mb, $T_0/\mu = 1$			**B)** $\sigma_0 = 100$ mb, $T_0/\mu = 0$		
b [fm]	$\langle n \rangle$	χ	b [fm]	$\langle n \rangle$	χ
0	33.0	10.1	0	35.8	23.9
2	31.7	9.72	2	34.3	22.9
4	28.1	8.61	4	30.2	20.1
6	23.0	7.05	6	24.0	16.0
8	15.9	4.87	8	16.3	10.9
10	8.16	2.50	10	8.23	5.49
12	2.15	0.66	12	2.18	1.45

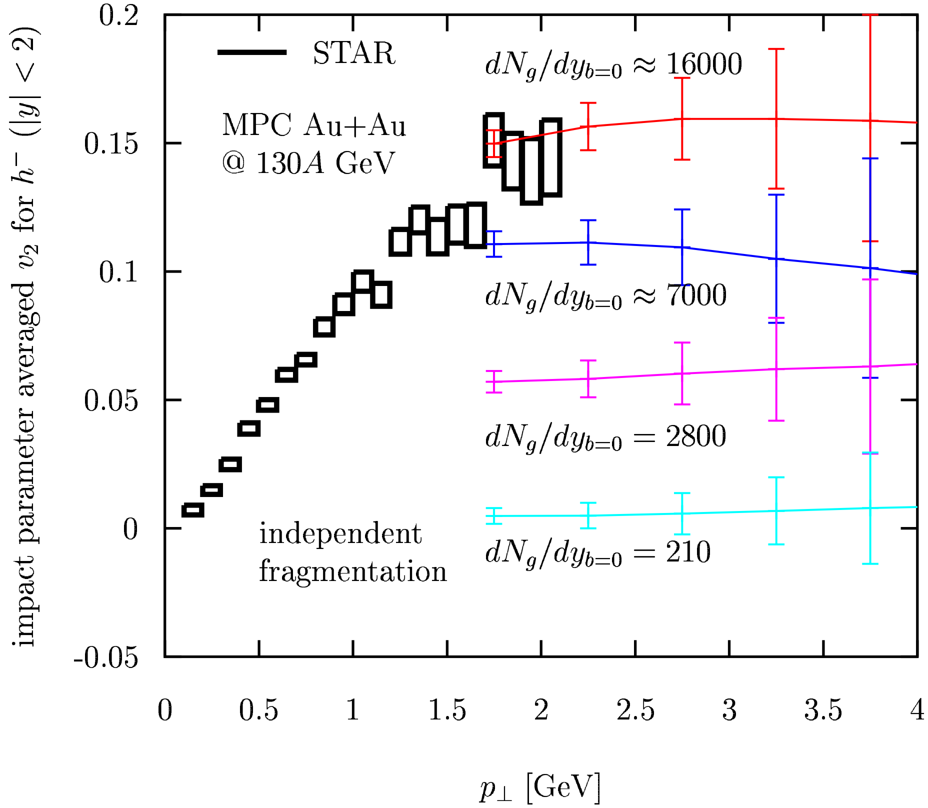
C) $\sigma_0 = 40$ mb, $T_0/\mu = 1$			**D)** $\sigma_0 = 40$ mb, $T_0/\mu = 0$		
b [fm]	$\langle n \rangle$	χ	b [fm]	$\langle n \rangle$	χ
0	13.4	4.11	0	13.7	9.13
2	12.9	3.95	2	13.2	8.80
4	11.4	3.49	4	11.6	7.73
6	9.26	2.84	6	9.38	6.25
8	6.37	1.95	8	6.44	4.29
10	3.23	0.99	10	3.27	2.18
12	0.86	0.26	12	0.86	0.57

E) $\sigma_0 = 3$ mb, $T_0/\mu = 1$			**F)** various, $b = 8$ fm			
b [fm]	$\langle n \rangle$	χ	σ_0 [fm]	T_0/μ	$\langle n \rangle$	χ
0	1.00	0.31	60	1.54	9.51	1.84
2	0.96	0.29	16	0	2.55	1.70
4	0.85	0.26	100	1.40	15.9	3.43
6	0.69	0.21	100	2.21	15.7	1.94
8	0.47	0.14	100	4.43	15.5	0.718
10	0.24	0.074				
12	0.064	0.020				

Table 1: Parameters and transport opacity for each transport solution computed via MPC for nucl-th/0104073.

Impact parameter averaged $v_2(p_T)$



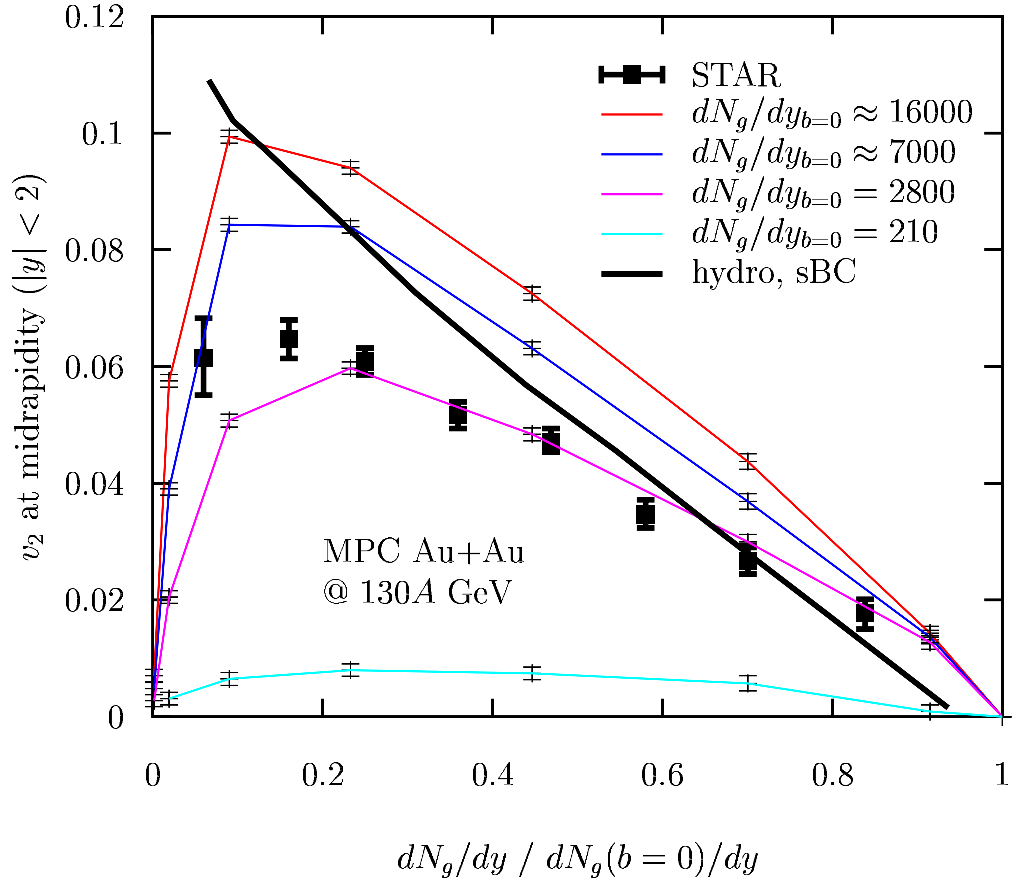


a) hadronization via parton-hadron duality

b) independent fragmentation

- **with pQCD $\sigma = 3$ mb, the data is reproduced for $dN_g/dy \approx 16000$**
- **no significant difference between the two hadronization models (indep. fragmentation is reliable only for $p_T > 2$ GeV)**
- **rapid expansion: $N_{coll} \sim 20 - 30$ does not ensure equilibrium**

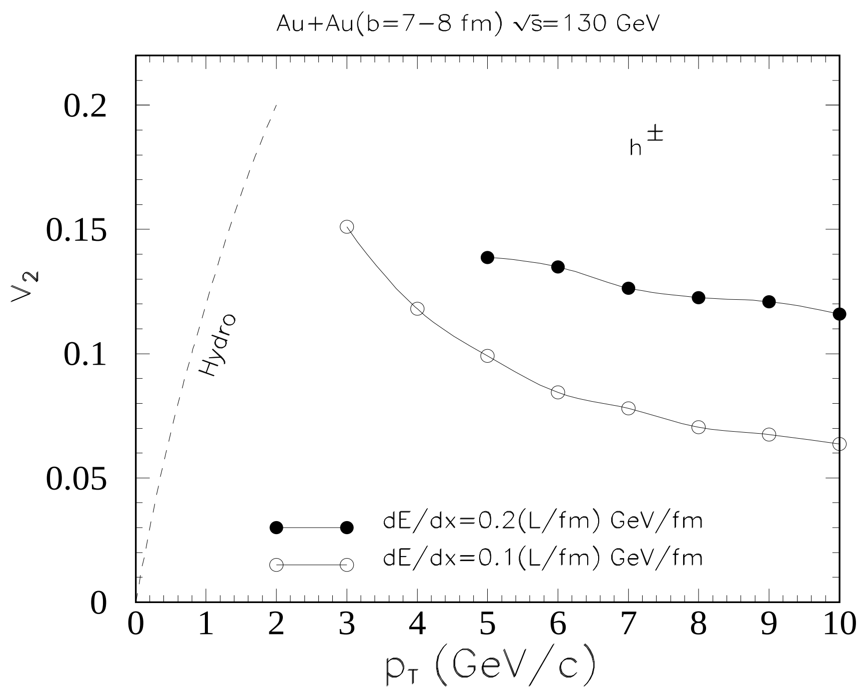
Impact parameter dependence of v2



- **for hadronization via local parton-hadron duality, same as** n_{ch}/n_{ch}^{max}
- **data can be reproduced down to** $n_{ch}/n_{ch}^{max} \sim 0.1 - 0.2$
  with $dN_g/dy \sim 3000 - 5000 < 16000$
- $v_2(b)$ **is especially sensitive to low-**p_T**, where our simple hadronization models are least reliable**

Saturation via inelastic energy loss

- **another possible explanation** (GVW model):
  pQCD inelastic parton energy loss
  + a parametrized, low-p_T "hydrodynamical" component



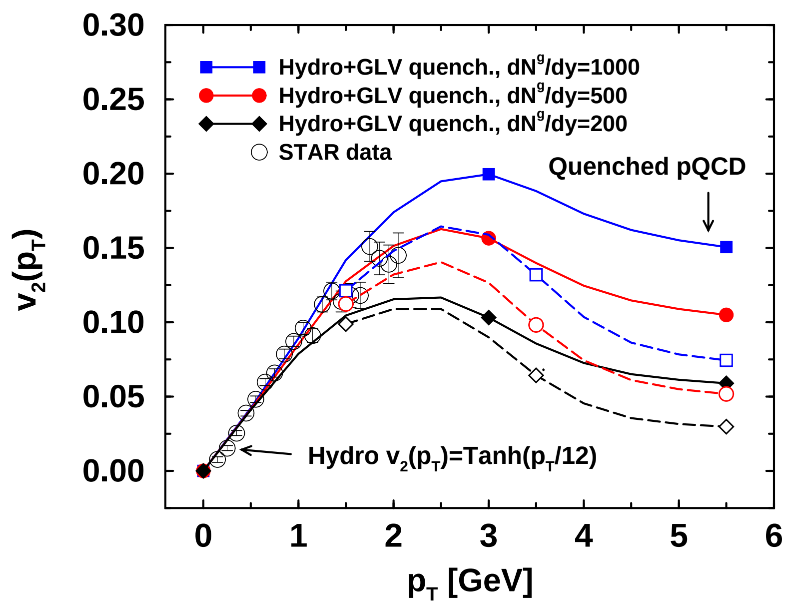


FIG. 4. The interpolation of $v_2(p_T)$ between the soft hydrodynamic [12] and hard pQCD regimes is shown for the same range of initial conditions as in Fig. 3. Solid (dashed) curves correspond to sharp cylindrical (diffuse Wood-Saxon) geometry presented in Fig. 2.

From nucl-th/0009019 (X.-N. Wang)

From nucl-th/0012092 (M. Gyulassy, I. Vitev, X.-N. Wang)

Conclusions

Classical Boltzmann theory is a convenient framework that interpolates between free streaming and ideal hydrodynamics. Therefore, it is especially suitable to study nonequilibrium phenomena.

Studies of transverse energy evolution and differential elliptic flow via the MPC parton transport technique indicate large deviations from equilibrium at RHIC, even for one order of magnitude larger gluon densities and cross sections than the pQCD estimates based on HIJING.

These extremely dense conditions were, on the other hand, found necessary to reproduce the saturation pattern of elliptic flow observed at RHIC. In particular, $\sim$ 80 times larger opacities ($\propto \sigma dN/dy$) than the HIJING estimate were needed to reproduce the STAR data.

It is expected that the elliptic flow data could be explained with more moderate opacities if inelastic parton energy loss was also taken into account. Unfortunately, no covariant algorithm yet exists to incorporate the simplest inelastic $3 \leftrightarrow 2$ process in on-shell parton cascades.

This talk is on the WWW at: http://nt3.phys.columbia.edu/people/molnard